

2021

ADVANCED BUSINESS MATHEMATICS — HONOURS

Sixth Paper

(A-32-A)

Full Marks : 50

*The figures in the margin indicate full marks.**Candidates are required to give their answers in their own words as far as practicable.*

Group-A

1. Answer the following questions:

2×5

(a) Find the range of the following function:

$$y = \frac{x^2}{1+x^2}$$

(b) If $\varphi(x) = 2^{mx+1}$, then show that $\varphi(a) \cdot \varphi(b) \cdot \varphi(c) = 4 \cdot \varphi(a + b + c)$.*Or,*

$$\text{If } f(x) = \frac{x-a}{x} + \frac{x}{x-b}, \text{ show that } f\left(\frac{a+b}{2}\right) = \frac{4ab}{a^2-b^2}.$$

(c) Draw the graph of the function:

$$\begin{aligned} y = f(x) &= x - 1, & x > 0 \\ &= -\frac{1}{2}, & x = 0 \\ &= x + 1, & x < 0 \end{aligned}$$

(d) Find: $\lim_{x \rightarrow 1} \frac{e^{\log x} - 1}{e^{x-1} - 1}$ *Or,*

$$\lim_{x \rightarrow 0} \frac{5^x - 4^x}{x}$$

(e) If $y = \frac{x-2}{x+2}$, then prove that $2x \frac{dy}{dx} = 1 - y^2$.*Or,*Differentiate $5x^4$ with respect to x^2 .

Group-B

2. Answer the following questions:

(a) Evaluate:

$$(i) \lim_{x \rightarrow 0} \frac{\sqrt{1+x} - \sqrt{1-x}}{x}$$

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(ii) $\lim_{x \rightarrow \infty} \frac{6-5x^2}{8x-15x^2}$ 3+3

(b) If $x = y^y y^{\dots \infty}$, show that $\frac{dy}{dx} = \frac{y(1-x \log y)}{x^2}$. 6

(c) If $2x + 3y = 4$, find the maximum or minimum value of xy . 6

Or,

Show that the maximum value of $\left(\frac{1}{x}\right)^x$ is $e^{1/e}$. 6

(d) Prove that $\begin{vmatrix} 2a & a-b-c & 2a \\ 2b & 2b & b-c-a \\ c-a-b & 2c & 2c \end{vmatrix} = (a+b+c)^3$ 6

Or,

Find the value of x when $\begin{vmatrix} x-1 & 1 & 1 \\ 1 & x+1 & -1 \\ -1 & 1 & x+1 \end{vmatrix} = 0$. 6

(e) Find A when $A^{-1} = \begin{bmatrix} 1 & 3 & 2 \\ -3 & -3 & -1 \\ 2 & 1 & 0 \end{bmatrix}$. 6

Or,

Solve the system of linear equations by matrix method: 6

$$2x + 3y + z = 17, \quad x - y + z = 3, \quad 3x + 2y - 2z = 4$$

Group-C

3. Answer the following questions:

(a) Evaluate (**any one**): 4×1

(i) $\int \frac{dx}{\sqrt{x+1} + \sqrt{x+2}}$

(ii) $\int \frac{x^3}{x^4+x^2-12} dx$

(b) A bag contains 6 white and 4 black balls. 2 balls are drawn at random. Find the probability that 6

(i) they are of same colour,

(ii) they are of different colour.

Or,

Two dice are rolled one after the other. Find the probability that the number on the first is smaller than the number of the second. 6
